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ON THE STRAIN ANALYSIS OF TECTONIC MOVEMENTS USING FAULT CROSSING GEODETIC SURVEYS

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ABSTRACT

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A generalized approach to the analysis of deformation surveys has been developed by the authors and utilized in the analysis of tectonic movements. The approach is applicable to any type of repeated geometrical measurements (geodetic and non-geodetic surveys), any type of deformations including rigid body displacements and strain, and geometrical configuration of the observation network. The approach is based on the least squares fitting of selected deformation models to the displacement field obtained from repeated observations of deformations. The approach consists of three basic processes: (1) preliminary identification of the deformation models; (2) estimation of the deformation parameters using a generalized mathematical model for the least squares fitting; and (3) diagnostic checking of the deformation models and the final selection of the "best" model based on global statistical tests and on calculated significance levels of the deformation parameters. A numerical example is given using survey data from four epochs of observations of a small geodetic network which was established across an active fault in the Peruvian Andes.

INTRODUCTION

Generally, in deformation measurements by geodetic methods, whether they are performed for monitoring engineering structures or ground subsidence in mining areas or tectonic movements, two basic types of geodetic networks are distinguished (Chrzanowski et al., 1981b):

(1) absolute networks in which some of the points are, or are assumed to be outside of the deformable body (object) thus serving as reference points (reference network) for the determination of absolute displacements of the object points;

(2) relative networks in which all the surveyed points are assumed to be located on the deformable body.

In the first case, the main problem of deformation analysis is to confirm the

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stability of the reference points and to identify the possible single point displacements caused, for instance, by local surface forces and wrong monumentation of the survey markers. Numerous approaches have been suggested by different authors to determine the stability of the reference points, for instance, methods developed by Pelzer (1974), Heck et al. (1977), Lazzarini (1977), Niemeier (1979), Polak (1981), and Van Mierlo (1981), just to mention a few. A comparison of some of the methods has been the subject of studies by a special committee (Chrzanowski et al., 1981b) of Commission 6 of F.I.G. (Fédération Internationale des Géomètres). Once the stable reference points are identified, the determination of the geometrical state of deformation of the deformable body is rather simple.

In the case of relative networks, deformation analysis is more complicated because, in addition to the possible single point displacements like in the reference network, all the points undergo relative movements caused by strains in the material of the body and by relative rigid translations and rotations of parts of the body if discontinuities in the material, for instance, tectonic faults, are present. The main problem in this case is to identify the deformation model, i.e. to distinguish, on the basis of repeated geodetic observations, between the deformations caused by the extension and shearing strains, by the relative rigid body displacements and by the single point displacements.

The geodetic monitoring of tectonic movements is usually done with relative geodetic networks unless extraterrestrial observations are included in the network. Thus the problem of identification of the deformation model is of primary importance in the analysis.

As far as strain analysis is concerned, the computational procedures are well known and have been applied in the analysis of tectonic movements for many years. A brief review of some basic works on the subject is given by Vanicek and Krakiwsky (1982). Recent papers by Margrave and Nyland (1980), Snay and Cline (1980), Savage et al. (1981), Prescott (1981), and Chrzanowski and Chen (1982) serve as a good sample of different approaches being used by different authors in the strain analysis of tectonic movements. The approaches can be classified into two basic types: (1) raw-observation approach; and (2) displacement approach. The first approach is based on the calculation of the strain components or their rates directly from differences in the repeated observations. In the second approach the strain components are calculated from differences in adjusted coordinates (displacements) of the geodetic points.

In both approaches, if the number of repeated observations or the number of derived displacements at discrete points is larger than the number of unknown deformation parameters, then the least squares fitting of a deformation model is performed to yield the parameters. If the same set of observables and weights is used in both approaches and the same deformation model is fitted either to the raw observations or to the derived displacements, the same solution for the deformation parameters would be obtained. The displacement approach is favoured by many authors, for instance by Bibby (1975), Brunner et al. (1980) and Chrzanowski and Chen (1982). Its main advantage is the possibility of using all observables in the strain analysis, even if they differ from one epoch of observations to another as long as they can be reduced to the same geodetic datum and can be used in the calculation of displacements. On the other hand, the raw-observation approach requires that the same observables and the same geometry of the network in each epoch be maintained and utilized. Some other advantages of the displacement approach are also mentioned below in this paper. However, the displacement approach may be inconvenient or even impossible to use if the geodetic network has configuration defects (lack of geometrical ties between the observables).

Usually more than one deformation model can be fitted to the observation data and then arises the question of which of the models is the "best". Other difficulties and ambiguities in the deformation analysis develop when different minimum constraints must be used in the least squares adjustments of individual epochs due to changes in the network geometry or when the geodetic observations must be combined with other types of observables such as tilt, strain, and alignment measurements.

In order to overcome the above problems the authors have elaborated on a generalized approach to the analysis of deformation surveys which, in a systematic, step-by-step manner, deals with the problems of analysis.

A description of the generalized approach and a numerical example are given in this paper. The example deals with the deformation analysis of a small geodetic network (Chrzanowski et al., 1981a) which was established across an active fault in the Peruvian Andes in 1975 as a joint project between the University of New Brunswick and the University of Alberta in Canada and the Geophysical Institute of Peru. The network has been measured four times annually, in 1975, 1976, 1977 and 1978. The 1978 survey was performed by a German team (Welsch, 1979).

BASIC CRITERIA OF THE GENERALIZED APPROACH

In developing the generalized approach the authors required that the following criteria be fulfilled:

(a) The approach should be applicable to any type of deformations, i.e. the same computational procedure should be used in the analysis of single point displacements in reference networks and in the analysis of rigid body displacements as well as in the determination of strain components in relative survey networks.

(b) The same approach should be used for one, two and three-dimensional survey data for the determination of deformation parameters either in a local domain or in a time domain if the strain rates are also required.

(c) Any type of survey data, i.e. not only geodetic (distances, angles, etc.) but also physical-mechanical measurements of tilts, strains, pendula deviations etc. should be utilized in a simultaneous analysis as long as the differences in the observed or quasi-observed (e.g. derived coordinates) quantities could be expressed as functions of relative displacements of the points at which the measurements were made. Hence, any further reference to observations is taken to include possible quasi-observations.

(d) The approach should be applicable to any geometrical configuration of the survey network including incomplete networks with configuration defects. In these, isolated observations which are not connected to other points of the network would be taken as long as approximate coordinates of all survey stations are given in the same coordinate system.

(e) Different minimum constraints (including inner constraints) could be used in the numerical processing of each epoch of observations as long as the same approximate coordinates of points are used in each of the epochs.

The proposed approach is based on the least squares fitting of a selected deformation model to the displacement field obtained from repeated observations of deformations at discrete points on the deformable body. Since more than one deformation model can be fitted to the given displacement field, the "best" model is selected on the basis of:

(i) an a priori knowledge, either actual or assumed, of the behaviour of the body,

(ii) a demonstrated deformation trend using the so called "Fredericton Approach" (Chrzanowski et al, 1981) which is based on an examination of the differences of observed quantities, or of quantities derived from adjusted coordinates, and an examination of plotted displacements in a local coordinate system which are obtained by using the "best" minimum constraints in the least squares treatment of the data,

(iii) global statistical tests and examination of residuals and calculated significance levels of the deformation parameters obtained through the process of fitting.

Thus, generally, the approach consists of three basic processes:

(1) Preliminary identification of the deformation model.

(2) Estimation of the deformation parameters.

(3) Diagnostic checking of the deformation models and the final selection of the "best" model.

These are briefly discussed below. Though the approach is applicable to 3-D analysis in local or time domains, the description of the procedures will refer only to a 2-D case in a local domain in order to facilitate a concise explanation.

PRELIMINARY IDENTIFICATION OF THE DEFORMATION MODEL

In each case of deformation analysis, when using the generalized approach the whole area covered by the deformation surveys is treated as a noncontinuous deformable body consisting of separate continuous deformable blocks. Thus the blocks may undergo relative rigid body displacements and rotations and each block may change its shape and dimensions. Thus in a 2-D analysis for each block the following deformation parameters in an X, Y coordinate system must be considered: two components (a_0 and b_0 or c_0 and g_0 etc.) of the rigid body displacement; a rotation parameter $\omega(x, y)$; two extension strain components, $\epsilon_x(x, y)$ and $\epsilon_y(x, y)$; and shearing strain, $\epsilon_{xy}(x, y)$.

For instance, in the case of single point displacement, the given point is treated as a separate block being displaced as a rigid body in relation to the undeformed block composed of the remaining points in the network.

The deformation of a block is fully described if a displacement function $\mathrm{d}(x, y)$ is given for the whole block. Once the components $\mathrm{d}x(x, y)$ and $\mathrm{d}y(x, y)$ of the displacement function are known, the strain components and the differential rotation may be calculated at any point from the well known infinitesimal strain-displacement relationships:

$$\epsilon_x = \frac{\partial}{\partial x}\mathrm{d}x, \qquad \epsilon_y = \frac{\partial}{\partial y}\mathrm{d}y, \qquad \epsilon_{xy} = \frac{1}{2}\left(\frac{\partial}{\partial x}\mathrm{d}y + \frac{\partial}{\partial y}\mathrm{d}x\right) \tag{1}$$

and:

$$\omega = \frac{1}{2}\left(\frac{\partial}{\partial x}\mathrm{d}y - \frac{\partial}{\partial y}\mathrm{d}x\right) \tag{2}$$

The displacement function is determined through a polynomial approximation of the displacement field. The general case would use the polynomials:

$$\mathrm{d}x = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + \ldots \tag{3}$$

$$\mathrm{d}y = b_0 + b_1x + b_2y + b_3xy + b_4x^2 + \ldots \tag{4}$$

Depending on the selected deformation model, some of the coefficients of the polynomials (3) and (4) are taken to be zero. Examples of typical deformation models are given below:

(a) *Single point displacement* or a *rigid body displacement* (Fig. 1a) of a group of points (say block B) with respect to a stable block (say block A); the deformation model is:

$$\mathrm{d}x_A = 0, \mathrm{d}y_A = 0, \mathrm{d}x_B = a_0 \text{ and } \mathrm{d}y_B = b_0 \tag{5}$$

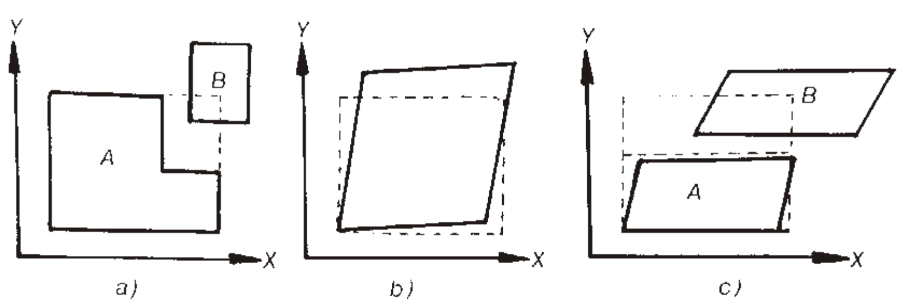


Fig. 1. Single deformation models: (a) rigid body displacement, (b) homogeneous strain and (c) rigid body displacement plus different homogeneous strains.

(b) *Homogeneous strain* (Fig. 1b) in the whole body without discontinuities; for the whole body, the linear deformation model is:

$$\mathrm{d}x = a_1 x + a_2 y \text{ and } \mathrm{d}y = b_1 x + b_2 y \tag{6}$$

which, after substituting eqs. 1 and 2 into eq. 6 becomes:

$$\mathrm{d}x = \epsilon_x x + \epsilon_{xy} y - \omega y \tag{7}$$

$$\mathrm{d}y = \epsilon_{xy} x + \epsilon_y y + \omega x \tag{8}$$

In the above, special attention must be paid to the rotation parameter ω which may have to be included in this model (and also in other models) even if the absolute orientation of the network is not available, depending on the computational constraints, as will be discussed below later on.

(c) *A deformable body with one discontinuity*, say between blocks A and B, with different linear deformations of each block plus a rigid body displacement of B with respect to A (Fig. 1c):

$$\mathrm{d}x_A = a_1 x + a_2 y$$

$$\mathrm{d}y_A = b_1 x + b_2 y$$

$$\mathrm{d}x_B = c_0 + c_1 x + c_2 y$$

$$\mathrm{d}y_B = g_0 + g_1 x + g_2 y$$

In the above case, components Δx_i and Δy_i of a total relative dislocation at any point i located on the discontinuity line between blocks A and B may be calculated as:

$$\Delta x_i = \mathrm{d}x_B(x_i, y_i) - \mathrm{d}x_A(x_i, y_i)$$

$$\Delta y_i = \mathrm{d}y_B(x_i, y_i) - \mathrm{d}y_A(x_i, y_i)$$

In general, a deformation model, if expressed by the displacement functions may be briefly written in the matrix form:

$$\boldsymbol{d}(x, y) = \mathbf{B}\boldsymbol{e} \tag{9}$$

where $\boldsymbol{d}$ is a vector of displacements, $\boldsymbol{e}$ is a vector of unknown deformation parameters (or the coefficients of the polynomials, $\mathbf{B}$ is a matrix containing elements which are functions of positions of the observation points, and of time if the rates of deformation are needed.

Usually, the actual deformation model is a combination of the above simple models or, if more complicated, it is expressed by non-linear displacement functions which require fitting of higher order polynomials. However, if no a priori knowledge on the expected model exists then the simplest reasonable model, one of the three listed above for instance, with the fewest unknown parameters, is selected for the preliminary fitting and testing. Here, the demonstrated deformation trend is of great help in the identification of the deformation model.

In order to determine the vector $\boldsymbol{e}$, the number of known displacements at discrete points must be, at least, equal to the number of unknown coefficients in the deformation model. If the number of known displacements is larger than that of the unknown coefficients, the deformation parameters are determined through the least squares fitting.

In the generalized approach, the deformation model may be fitted either to differences of coordinates (displacements) estimated from repeated observations of k epochs or directly to differences of observed quantities, for instance distances, angles, strains, tilts, etc.

In the latter instance one can either:

(1) transform all the observed quantities into changes of coordinates, even in the case of there being configuration defects in the network; or

(2) express each difference of the observed quantities in terms of the components $\mathrm{d}x(x, y)$ and $\mathrm{d}y(x, y)$ of the selected deformation model.

In the first case, if the observables belong to a complete geodetic network the coordinates and derived displacements are estimated through the least squares adjustment of individual epochs. If the observables are not properly connected together (configuration defects) then differences of individual observations may be transformed into differences of coordinates by introducing local constraints. For instance, a change $\mathrm{d}l_{ij}$ in a measured distance l_{ij} between points i and j with known approximate coordinates x^0 and y^0 may be transformed into relative displacements $\mathrm{d}x_j$ and $\mathrm{d}y_j$ with respect to point i (or vice versa) by constraining (fixing) point i and azimuth α_{ij}^0 calculated from the approximate coordinates, thus obtaining: $\mathrm{d}x_j = \mathrm{d}l_{ij} \cdot \sin(\alpha_{ij}^0)$ and $\mathrm{d}y_j = \mathrm{d}l_{ij}\cos(\alpha_{ij}^0)$. This could be regarded as a general approach using displacements. The calculated $\mathrm{d}x_j$ and $\mathrm{d}y_j$ are used later in the process of fitting the deformation model into the "observed" displacements (quasi-observables).

In the second case, the same difference $\mathrm{d}l_{ij}$ may be expressed in terms of the displacement functions $\mathrm{d}x(x, y)$ and $\mathrm{d}y(x, y)$ using the Taylor's series expansion:

$$\mathrm{d}l_{ij} = \frac{x_j^0 - x_i^0}{l_{ij}}\,\mathrm{d}x(x_j, y_j) - \frac{x_j^0 - x_i^0}{l_{ij}}\,\mathrm{d}x(x_i, y_i) + \frac{y_j^0 - y_i^0}{l_{ij}}\,\mathrm{d}y(x_j, y_j) - \frac{y_j^0 - y_i^0}{l_{ij}}\,\mathrm{d}y(x_i, y_i)$$

in which the functions $\mathrm{d}x(x, y)$ and $\mathrm{d}y(x, y)$ are replaced by the proper portions of the polynomials (3) and (4) depending on the selected deformation model. Thus in short, the vector of differences $\boldsymbol{dl}$ of any type of observation may be expressed in terms of the deformation model by:

$$\boldsymbol{dl} = \mathbf{AB}\boldsymbol{e} \tag{10}$$

where **A** is the design matrix of the observation equations. This is the raw observation approach.

If all the elements of the $\boldsymbol{dl}$ vector are really quasi-observed displacement components, then the **A** matrix is the identity and (10) reduces to the same form as (9).

It does not matter which of the two above approaches is followed in the preparation of the observation data for the polynomial approximation of the displacement field because the generalized approach can handle either or both types of fitting in a simultaneous solution. However, if the whole observation network or, at least, a large portion of it is a complete geodetic network (without configuration defect) then the first (displacement) approach is recommended so that the displacements of all the points are derived from the observed quantities through the least squares estimation of the coordinates in each epoch. The least squares adjustment allows for screening of the observation data for outliers and for statistical evaluation of the quality of the observations. Besides, the displacement approach gives a better picture of the deformation trend than the raw observation approach through the above mentioned examination of the display of the plotted relative displacements of points.

The process of the least squares determination of the $\boldsymbol{e}$ parameters is based on the null hypothesis:

$$H_0\colon E(\boldsymbol{x}_0) = \boldsymbol{\xi} = E(\boldsymbol{x}_i - \mathbf{A}_i\mathbf{B}_i\boldsymbol{e})$$

with an alternative hypothesis:

$$H_A\colon E(\boldsymbol{x}_0) = \boldsymbol{\xi} \neq E(\boldsymbol{x}_i - \mathbf{A}_i\mathbf{B}_i\boldsymbol{e})$$

with $E(.)$ being the expected value, $\boldsymbol{x}_i$ being the vector of coordinates (if all observed quantities have been transformed into coordinates) or other observed quantities in epoch i $(i = 0, 1, \ldots, k)$ and $\boldsymbol{\xi}$ being a vector of unknown constants.

As mentioned before, if all the observations of deformations have been transformed into differences of coordinates, then the vectors $\boldsymbol{x}_i$ contain only coordinates of points (quasi-observables) and matrix **A** becomes the identity matrix. For simplicity of derivation, the matrix **A** is omitted in the further discussion (it may be treated as a part of a new **B** matrix).

The above null hypothesis leads to the generalized mathematical model:

$$\boldsymbol{x}_0 + \boldsymbol{v}_0 = \boldsymbol{\xi} \tag{11}$$

$$\boldsymbol{x}_i + \boldsymbol{v}_i = \boldsymbol{\xi} + \mathbf{B}_i\boldsymbol{e} \tag{12}$$

where $\boldsymbol{v}_i$ $(i = 0, 1, \ldots, k)$ is a vector of residuals.

In order to solve eqs. 11 and 12 for $\boldsymbol{e}$, the population of the $\boldsymbol{x}_i$ vectors must be the same in each epoch of the simultaneously compared observations even if the actual number n and type of observations differ from one epoch to another. This is achieved by placing artificial observations (for instance their estimated approximate values) with weights equal to zero in place of the missing observations in the x_i vectors. The least squares criterion applied to the above model with $k+1$ epochs of observations leads to the following normal equations:

$$\begin{pmatrix} \sum_0^k \mathbf{P}_i & \sum_1^k \mathbf{P}_i\mathbf{B}_i \\ \sum_1^k \mathbf{B}_i^{\mathrm{T}}\mathbf{P}_i & \sum_1^k B_i^{\mathrm{T}}\mathbf{P}_i\mathbf{B}_i \end{pmatrix} \begin{pmatrix} \boldsymbol{\xi} \\ \boldsymbol{e} \end{pmatrix} = \begin{pmatrix} \sum_0^k \mathbf{P}_i\boldsymbol{x}_i \\ \sum_1^k \mathbf{B}_i^{\mathrm{T}}\mathbf{P}_i\boldsymbol{x}_i \end{pmatrix}$$

where the $\mathbf{P}_i$ are weight matrices of the observations (quasi-observations) and are singular in general.

These normal equations are singular with rank defect $\mathrm{d}(\Sigma_0^k \mathbf{P}_i) = \mathbf{d}$, so that no unique solution exists. Eliminating $\boldsymbol{\xi}$ from the equation yields invariant deformation parameters $\boldsymbol{e}$ for any choice of the generalized inverse $(\Sigma_0^k \mathbf{P}_i)^-$ provided $\mathbf{B}_i$ is selected properly:

$$\hat{\boldsymbol{e}} = \left\{\sum_1^k \mathbf{B}_i^{\mathrm{T}}\mathbf{P}_i\mathbf{B}_i - \sum_1^k \mathbf{B}_i^{\mathrm{T}}\mathbf{P}_i\left(\sum_0^k \mathbf{P}_i\right)^- \sum_0^k \mathbf{P}_i\mathbf{B}_i\right\}^{-1} \left\{\sum_1^k \mathbf{B}^{\mathrm{T}}\mathbf{P}_i\boldsymbol{x}_i - \sum_1^k \mathbf{B}_i^{\mathrm{T}}\mathbf{P}_i\left(\sum_0^k \mathbf{P}_i\right)^- \sum_0^k \mathbf{P}_i\boldsymbol{x}_i\right\} =$$

$$= \mathbf{Q}_c \cdot \boldsymbol{w} \tag{13}$$

The covariance $\hat{\mathbf{C}}_e = \hat{\sigma}_0^2\mathbf{Q}_e$ with $\hat{\sigma}_0^2$, the variance factor, being estimated from preadjustment evaluation of the observations (e.g. through a simultaneous least squares adjustment of coordinates for $k+1$ epochs).

If only two epochs of observationd are analysed at the same time and in each epoch the elements of the x_i vectors were obtained using the same minimum constraints in both epochs then the equations (11) and (12) may be subtracted from each other leading to a simpler model:

$$\boldsymbol{d} + \boldsymbol{v}_d = \mathbf{B}\boldsymbol{e} \tag{14}$$

where $\boldsymbol{d} = \boldsymbol{x}_i - \boldsymbol{x}_0$ and $\boldsymbol{v}_d = \boldsymbol{v}_i - \boldsymbol{v}_0$ with the covariance matrix:

$$\mathbf{C}_d = \hat{\sigma}_0^2\left(\mathbf{Q}_{x_i} + \mathbf{Q}_{x_0}\right) = \hat{\sigma}_0^2\mathbf{Q}_d \tag{15}$$

where $\hat{\sigma}_0^2$, the weighted variance factor, and the cofactor matrices $\mathbf{Q}$ are obtained from the parametric least squares estimations of the $\boldsymbol{x}_i$ vectors.

The solution is then obtained from:

$$\hat{\boldsymbol{e}} = \left(\mathbf{B}^{\mathrm{T}}\mathbf{Q}_d^{-1}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_d^{-1}\boldsymbol{d} \tag{16}$$

with a covariance matrix:

$$\hat{\mathbf{C}}_e = \hat{\sigma}_0^2\left(\mathbf{B}^{\mathrm{T}}\mathbf{Q}_d^{-1}\mathbf{B}\right)^{-1} = \hat{\sigma}_0^2\mathbf{Q}_e \tag{17}$$

Thus the solution (16) which represents the aforementioned displacement approach to strain analysis is a special case of the generalized approach.

In the above, caution should be exercised in the computations if the displacements $\boldsymbol{d}$ and $\mathbf{Q}_d$ are derived from a least squares solution with minimum constraints. For instance, in the adjustment of a trilateration or triangulateration (mixed triangulation and trilateration) network, a point P_i and the direction from P_i to a second point P_j act as constraints on the degrees of freedom of the configuration. This is imposed by fixing the coordinates (x_i, y_i) of P_i which is then taken as the origin of the local coordinate system (Chrzanowski and Chen, 1982) and by assigning a very small variance to the azimuth α_{ij}. Consequently, when comparing the coordinates under the same constraints at two epochs, the displacement of P_j is confined to occur in the direction of the azimuth α_{ij} so that:

$$\mathrm{d}x_j/\mathrm{d}y_j = \tan(\alpha_{ij})$$

If, for instance, the displacement field is approximated by the model expressed by eqs. 7 and 8 then the observation equations for the displacement components of point P_j are:

$$\mathrm{d}x_j + v_{xj} = \epsilon_x x_j + \epsilon_{xy} y_j - \omega y_j$$

$$\mathrm{d}y_j + v_{yj} = \epsilon_{xy} x_j + \epsilon_y y_j + \omega x_j$$

If the deformation parameters are estimated from (16), then the very small variance of α_{ij} constrains these to have the relation:

$$\frac{\epsilon_x x_j + \epsilon_{xy} y_j - \omega y_j}{\epsilon_{xy} x_j + \epsilon_y y_j + \omega x_j} = \tan(\alpha_{ij})$$

so that:

$$\omega = \tfrac{1}{2}(\epsilon_x - \epsilon_y)\sin(2\alpha_{ij}) + \epsilon_{xy}\cos(2\alpha_{ij})$$

Thus, the variation associated with the change in minimum constraints is absorbed by the change in the value of ω, rendering the values of the other strain parameters invariant. If ω were omitted (considered as being zero) in the model, as it is done by some authors when no absolute orientation of the network is available (no ties to external reference points), then the calculated strain parameters would vary with the choice of the minimum constraints. The same applies to other deformation models, whenever the constrained direction α_{ij} is within the deformed part of the investigated body. For instance, in the case of rigid body displacements as shown in Fig. 1a, if point i of the constrained direction would be in block A and point j in block B, then eqs. 5 of the deformation model would be written in the form:

$$\mathrm{d}x_A = -\omega y, \quad \mathrm{d}y_A = \omega x, \quad \mathrm{d}x_B = a_0 - \omega y, \text{ and } \mathrm{d}y_B = b_0 + \omega x$$

Otherwise, the values of the parameters a_0 and b_0 would be dependent on the choice of the direction α_{ij} to be constrained. The rotation parameter ω in the above cases plays the role of a nuisance parameter without any practical meaning. However, if $\mathbf{Q}_d$ is calculated in a solution using inner constraints and if $\mathbf{Q}_d^+$, the pseudoinverse of $\mathbf{Q}_d$, is used in eq. 16, the omission of ω is justified when no external orientation of the network is included in the observables.

As is usually done in the least squares estimation process, the quadratic form of the residuals is employed in a global test on the appropriateness of the model. The null hypothesis is that the variance factor $\hat{\sigma}_{0e}^2$, estimated by the adjustment yielding the $\hat{e}$ is the same as the a priori value, $\hat{\sigma}_0^2$, from the combination of any pair epoch adjustments for $\hat{x}_i$:

$H_0 : \hat{\sigma}_{0e}^2 = \hat{\sigma}_0^2$ versus $H_A : \hat{\sigma}_{0e}^2 \neq \hat{\sigma}_0^2$

The $\hat{\sigma}_{0e}^2$ is calculated, for eqs. 11 and 12, by:

$$\hat{\sigma}_{0e}^2 = \frac{\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}}{f_e} = \frac{\sum_0^k \mathbf{v}_i^{\mathrm{T}}\mathbf{P}_i\mathbf{v}_i}{f_e} \tag{18}$$

where $\mathbf{P}$ is the weight hypermatrix:

$$\mathbf{P} = \begin{pmatrix} \mathbf{P}_0 & & & \\ & \mathbf{P}_1 & & \\ & & \ddots & \\ & & & \mathbf{P}_k \end{pmatrix}$$

and f_e degrees of freedom is the rank of $\mathbf{P}$ minus $(u - d)$ where u is the number of unknowns. The statistic then becomes:

$$T^2 = \frac{\hat{\sigma}_{0e}^2}{\hat{\sigma}_0^2} \tag{19}$$

This is compared against the critical value $F(f_e, f, \alpha)$ with f being the degrees of freedom associated with the estimation of $\hat{\sigma}_0^2$. The null hypothesis is rejected at the α level of significance if the statistic exceeds the critical value. The simpler case, eq. 14, leads to the statistic:

$$T^2 = \frac{\hat{\sigma}_{0e}^2}{\hat{\sigma}_0^2} = \frac{\mathbf{v}_d^{\mathrm{T}}\mathbf{Q}_d^{-1}\mathbf{v}_d}{(2p - d - u)\hat{\sigma}_0^2} \tag{20}$$

Now, the null hypothesis is rejected if the statistic exceeds the critical value, i.e., if $T^2 > F(2p - d - u, f, \alpha)$. The degrees of freedom for the numerator is $(2p - d - u)$ in which there are p points in the network, having a defect of d and u is dim $\{e\}$. If the null hypothesis is not rejected, then the model is acceptable.

The significance of the individual parameters is revealed by considering each in its standardized form:

$$\tilde{e}_i = \frac{\hat{e}_i}{\sqrt{\hat{\sigma}_{ei}^2}} = \frac{\hat{e}_i}{\sqrt{(\hat{\sigma}_0^2 q_{ei})}}$$

with the q_{ei} being the ith diagonal element of $\mathbf{Q}_e$. The level of significance, α_{ei}, is obtained from:

$$\tilde{e}_i = \sqrt{F(1, f, \alpha)} \tag{21}$$

Or, a group of u_i parameters, $\hat{\boldsymbol{e}}_i$ which is a subset of $\hat{\boldsymbol{e}}$, may be considered with their quadratic form in the numerator of:

$$T^2 = \frac{\hat{\boldsymbol{e}}_i^{\mathrm{T}} \mathbf{Q}_{ei}^{-1} \hat{\boldsymbol{e}}_i}{u_i \hat{\sigma}_0^2} = F(u_i, f, \alpha) \tag{22}$$

from which the level of significance α is obtained. The numerator has degrees of freedom of u_i which is the rank of $\mathbf{Q}_{ei}$, a submatrix of $\mathbf{Q}_e$ from (13) or (17).

Because the behaviour of the deformable body is usually not completely known, there is often more than one possible model that may be appropriate (accepted by the global test). The choice of the "best" of the acceptable models has regard both for statistical significance of the parameters and for physical appropriateness, which would have justified consideration a priori.

SUMMARY OF PROCEDURES WHEN USING THE GENERALIZED APPROACH

(1) Adjustment (if applicable) of each epoch of observations for the evaluation of accuracy of observations and for the detection of outliers using $\tau_{\max}$ criterion (Pope, 1976), for instance.

(2) Comparison of two epochs; use of the Fredericton approach to the analysis of relatively stable points; choice of "best" minimum constraints to yield the "best" pattern of displacements.

(3) Choice of deformation model based on a priori considerations and the displacement pattern.

(4) Estimation of deformation parameters and their covariance.

(5) Global test on the deformation model; testing groups of parameters or an individual parameter for significance.

(6) Comparison of models and choice of "best" model.

(7) Graphical display of the selected model or models using simple rectangular blocks to represent the zones of the deformable body.

EXAMPLE

Description of the survey data

The aforementioned Peruvian network, known as the Huaytapallana network (Chrzanowski et al., 1981b), or as Huancayo network (Margrave and Nyland, 1980) has been analyzed by the authors using the described approach.

The network (Fig. 2) is located in the Huaytapallana mountain range of the

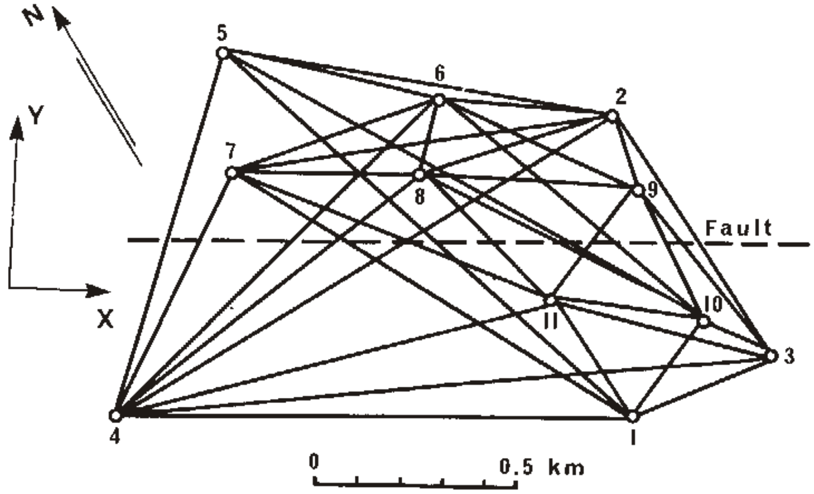


Fig. 2. Huaytapallana network.

Peruvian Andes at an average elevation of 4500 m and crosses a reverse fault which was activated by an earthquake in 1969 (Deza, 1971). At that time, a vertical displacement of 1.6 m and a horizontal strike-slip motion of 0.7 m were recorded. When designing the network and the survey (Nyland et al., 1979; Chrzanowski et al., 1981a) no additional information on the expected deformations was available. The goal of this microgeodetic survey was to detect relative rigid body displacements of groups of points on both sides of the fault with a standard deviation in the order of 3 mm, or better, and to determine strain components with standard deviations in the order of $3 \cdot 10^{-6}$.

Due to difficult topographic conditions only horizontal surveys were carried out. For economical reasons only standard surveying equipment could be used. The

TABLE I

Type and standard deviations of observations, Huaytapallana Network

Type of observations *	1975	1976	1977	1978
Angles				
N	73	81	2	–
σ	2.6″	2.2″	3.9″	
Directions				
N	–	–	91	91
σ			2.8″	2.5″
Distances				
N	60	65	74	70
σ (mm)	4.0	3.5	2.7	5.5

* N = number of observations.

eleven points of the network were monumented in rock outcrops using brass markers. Table I summarizes the type and number of observables and their estimated standard deviations in four epochs of observations.

Identification of the deformation models

Due to the lack of geophysical information on the expected deformation of the investigated area, several simple models were selected for further testing starting with the simplest assumption that no deformations had taken place ($dx = 0$ and $dy = 0$ for the whole area), followed by a model of the rigid body displacement along the fault line of the northern vs. southern parts of the network, then a homogeneous strain model for the whole area, and then different homogeneous models on both sides of the fault. In addition, some more complicated models had been selected on the basis of the preliminary trend analysis using the aforementioned Fredericton approach.

Figure 3 gives an example of the trend analysis based on the examination of displacements derived from the least squares adjusted coordinates in epochs 1978–1977. In this pair of epochs points *4* and *11* indicated a movement separate from the remaining portion of the network, thus indicating the possibility of an additional discontinuity running from the fault line through the vicinity of point *1* and isolating these two points. This trend had been confirmed also by the examination of epochs 1976–1975.

An examination of differences of angles and distances derived from the adjusted coordinates of four epochs led also to the suspicion that point *2* was unstable. Therefore, additional deformation models had been selected for further investigation which took under consideration the possibility of the separate rigid body displace-

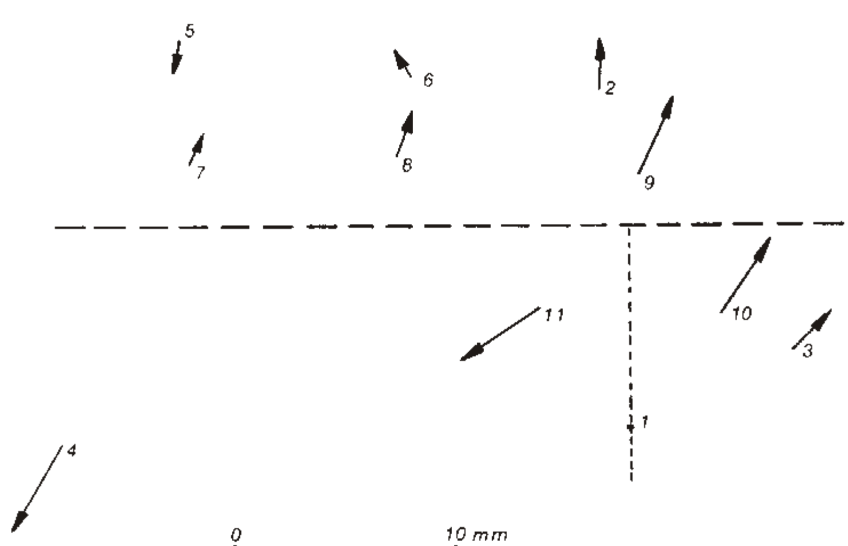


Fig. 3. Relative displacements of points 1978–1977 in the Huaytapallana network.

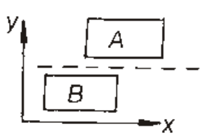
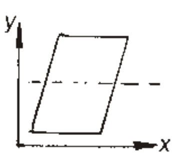
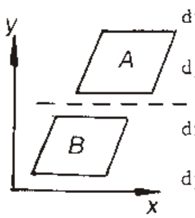
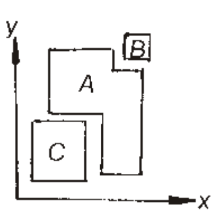
ments of points *4* and *11* as one block and point *2* as another block with respect to the remaining portion of the network.

Results and selection of the "best" model

The observation data allowed the least squares adjustments of coordinates using the same minimum constraints in each pair of epochs. Therefore, solutions for $\hat{e}$ and C_e of the selected deformation models could be obtained using the eqs. 14 17. A

TABLE II

Deformation models and global tests, Huaytapallana Network

Model No.	Deformation Model	Global Tests	
		Epochs	$T^2 \lessgtr F_{95\%}$
1.	No deformation $dx = 0$ $dy = 0$	'76–'75 '77–'76 '78–'77 '78–'75	1.48 < 1.63 1.38 < 1.63 1.44 < 1.63 0.80 < 1.63
2.	 $dx_A = 0 \quad dx_B = a_o$ $dy_A = 0 \quad dy_B = b_o$	'76–'75 '77–'76 '78–'77 '78–'75	1.35 < 1.69 1.39 < 1.68 0.92 < 1.68 0.60 < 1.69
3.	 $dx = \varepsilon_x x + \varepsilon_{xy} y - \omega y$ $dy = \varepsilon_{xy} x + \varepsilon_y y + \omega x$	'76–'75 '77–'76 '78–'77 '78–'75	1.32 < 1.71 1.35 < 1.71 1.15 < 1.71 0.68 < 1.71
4.	 $dx_A = \varepsilon_x x + \varepsilon_{xy} y - \omega y$ $dy_A = \varepsilon_{xy} x + \varepsilon_y y + \omega x$ $dx_B = a_o + \varepsilon_x x + \varepsilon_{xy} y - \omega y$ $dy_B = b_o + \varepsilon_{xy} x + \varepsilon_y y + \omega x$	'76–'75 '77–'76 '78–'77 '78–'75	1.43 < 1.76 1.51 < 1.76 0.91 < 1.76 0.67 < 1.76
5.	 $dx_A = 0 \quad dx_B = a_o$ $dy_A = 0 \quad dy_B = b_o$ $dx_C = c_o$ $dy_C = g_o$	'76–'75 '77–'76 '78–'77 '78–'75	0.44 < 1.73 1.00 < 1.73 0.37 < 1.73 0.69 < 1.73

total of eight deformation models were fitted to the data and examined. Table II illustrates five models which had been accepted at the 95% confidence level by the global statistical test (20). Table III summarizes the results of the least squares estimation of the deformation parameters and their probability levels $Pr\%$ calculated from (21) and (22) as $Pr\% = (1 - \alpha)\ 100$.

As one can see, the first model, "no-deformation", had passed the global tests at the 95% confidence level. However, after a close examination of the results, model No. 5 had been accepted as the "best" on the basis of the indicated trend of displacements, high confidence levels (> 99%) of its parameters and very favourable

TABLE III

Results of the least squares fitting of deformation models, Huaytapallana Network

Model No.	Deform. parameters	1976–1975		1977–1976		1978–1977		1978–1975	
		e_i	Pr%	e_i	Pr%	e_i	Pr%	e_i	Pr%
1.	–	–		–		–		–	
2.	a_o [mm]	1.8	89%	-1.6	89%	-3.4	99%	-3.2	97%
	b_o [mm]	2.2	94%	0.0	1%	-3.7	99%	-1.8	75%
	$(a_o^2+b_o^2)^{1/2}$	2.9	93%	1.6	73%	4.2	>99%	3.6	93%
3.	ε_x [μstrain]	-1.6	86%	-0.4	33%	1.6	81%	-0.4	21%
	ε_y [μstrain]	-2.6	80%	0.8	33%	3.8	89%	2.0	55%
	ε_{xy} [μstrain]	-1.6	91%	1.7	96%	2.1	94%	2.5	95%
4.	ε_x [μstrain]	-1.7	88%	-0.4	37%	2.0	90%	-0.2	11%
	ε_y [μstrain]	-0.4	10%	2.5	54%	-2.5	41%	-1.8	28%
	ε_{xy} [μstrain]	-0.8	36%	2.1	84%	-1.0	38%	1.1	39%
	a_o [mm]	1.3	51%	0.7	28%	-4.8	95%	-2.2	62%
	b_o [mm]	1.6	54%	1.2	46%	-4.4	88%	-2.7	63%
5.	a_o	-2.6	96%	3.7	>99%	0.8	37%	0.0	1%
	b_o	-2.9	94%	1.2	58%	-0.4	16%	-1.5	54%
	$(a_o^2+b_o^2)^{1/2}$	3.9	>99%	3.9	>99%	0.8	15%	1.5	26%
	c_o [mm]	2.7	99%	0.3	22%	-4.8	>99%	-1.4	65%
	g_o [mm]	1.2	63%	-0.0	1%	-4.0	99%	-2.9	91%
	$(c_o^2+g_o^2)^{1/2}$	3.0	99%	0.3	4%	6.2	>99%	3.2	88%

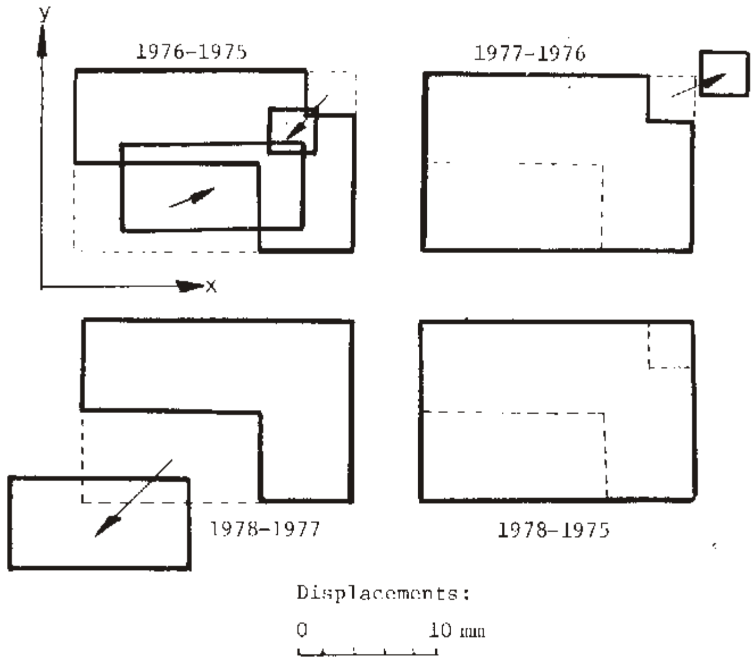


Fig. 4. The "best" deformation models for different epochs of observations, Huaytapallana network.

global tests. Figure 4 gives a graphical display of the estimated deformations of the area covered by the survey network using the "best" deformation model. In this model, block *C* contains points *4* and *11* of the network and block *B* contains point *2*. Most probably, the estimated relative displacements of the blocks, particularly of point *2*, are of a local, non-tectonic nature, such as surface movements of the marked survey points. However, the movement of points *4* and *11* as one block, if not coincidental may indicate an additional crustal discontinuity and an action of tectonic forces. Only additional future remeasurements of the network will allow for more concrete conclusions.

It is interesting to note that the survey data of the Huaytapallana network were also analyzed by Margrave and Nyland (1980). They considered only the homogeneous strain model in their analysis. They concluded that between 1975 and 1976 the area of the survey experienced a left lateral shear strain of about -3 microstrains which was possibly associated with tensional straining perpendicular to the fault. From 1976 to 1978 a right lateral shear strain of about $+3$ microstrains occurred in this area and was associated with a probable tensional straining parallel to the fault. In their analysis, Margrave and Nyland used the raw-observation approach, thus not being able to utilize all the observables which differed from one epoch to another. This may partially explain the numerical deviations from the values shown for model No. 3 in Table III. However, the overall conclusions would be in agreement with the results of two papers if the model No. 3 were accepted as the "best" model. Since that was not the case, the comparison of the final results indicates how important it is to follow the proposed generalized approach to analysis with a careful examination of more than just one deformation model.

CONCLUSIONS

Deformation surveys require a very complex and sophisticated analysis in order to avoid a misinterpretation of the geometrical status and behaviour of the deformable body. The generalized approach developed gives a new mathematical tool to handle the complex deformation analysis in a comparatively easy way with full utilization of statistical testing and logical judgement in a systematic, step-by-step manner. Only a brief introduction to the new approach has been given in this paper. A full evaluation of the generalized approach with additional numerical examples will be given in a separate technical report (Chen, 1983).

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REFERENCES

Bibby, H.M., 1975. Crustal strain from triangulation in Marlborough, New Zealand. Tectonophysics, 29: 529–540.

Brunner, F.K., Coleman, R. and Hirsch, B., 1980. Investigation of the significance in incremental strain values near Palmdale, California. Aust. J. Geod., Photogramm. Surv., 33: 57–74.

Chen, Y.Q., 1983. Analysis of Deformation Surveys, Rep. No. 94. Dep. Surv. Eng., Univ. of New Brunswick–A Generalized Method..

Chrzanowski, A. and Chen, Y.Q., 1982. Strain analysis of crustal deformations from repeated geodetic surveys. Proc. Conf. of South African Surveyors (CONSAS' 82), 7th, Johannesburg, 1982.

Chrzanowski, A., Nyland, E., Dennler, M. and Szostak, A., 1981a. Microgeodetic networks in monitoring tectonic movements. Proc. Int. Symp. on Deformation Measurements by Geodetic Methods, 2nd, Bonn, 1978. Wittwer, Stuttgart.

Chrzanowski, A. with contributions by members of the F.I.G. "ad hoc" committee, 1981b. A comparison of different approaches into the analysis of deformation measurements. F.I.G. Int. Congr., 16th, Pap. No. 602.3, Montreux.

Deza, E., 1971. The Pariahuanca earthquakes, Huancayo, Peru: July-October 1969. In: Recent Crustal Movements. R. Soc. N. Z., Bull., 9: 77–83.

Heck, B., Kuntz, E. and Meier-Hirmer, B., 1977. Deformations-Analyse mittels relativer Fehlerellipsen. Allgem. Vermess. Nachr., 84(3): 78–87.

Lazzarini, T., 1977. Geodezyjne pomiary przemieszczen budowli i ich otoczenia. Panstw. Przeds. Wyd. Kartogr., Warsaw.

Margrave, G.F. and Nyland, E., 1980. Strain from repeated geodetic surveys by generalized inverse methods. Can. J. Earth Sci., 17: 1020–1029.

Niemeier, W., 1979. Zur Kongruenz mehrfach beobachteter geodätischer Netze. Wiss. Arb. Fachrichtung Vermessungsw. Univ. Hannover, 88.

Nyland, E., Chrzanowski, A., Deza, E., Margrave, G., Dennler, M. and Szostak, A., 1979. Measurement and analysis of ground movement using microgeodetic networks on active faults. Geofis. Int. (Mex.), 18 (1).

Pelzer, H., 1974. Neuere Ergebnisse bei der statistischen Analyse von Deformationsmessungen. F.I.G. Congr., 14th, Washington, D.C., Pap. 608-3.

Polak, M., 1981. Examination of the stability of reference points in distance and combined angle-distance networks. Proc. Int. Symp. on Deformation Measurements by Geodetic Methods, 2nd, Bonn, 1978. Wittwer, Stuttgart.

Pope, A.J., 1976. The statistics of residuals and the detection of outliers. U.S. Dep. Commerce, NOAA Tech. Rep., NOS 65 NGS1.

Prescott, W.H., 1981. The determination of displacement fields from geodetic data along a strike-slip fault. J. Geophys. Res., 86: 6067–6072.

Savage, J.C., Lisowski, M. and Prescott, W.H., 1981. Geodetic strain measurements in Washington. J. Geophys. Res., 86: 4929–4940.

Snay, R.A. and Cline, M.W., 1980. Crustal movement investigations at Tejon Ranch, California. U.S. Dep. Commerce, NOAA Tech. Rep., NOS 87, NGS 18.

Vanicek, P. and Krakiwsky, E., 1982. Geodesy: the Concepts. North-Holland, Amsterdam.

Van Mierlo, J., 1981. A testing procedure for analysing geodetic deformation measurements. Proc. Int. Symp. on Deformation Measurements by Geodetic Methods, 2nd, Bonn, 1978. Wittwer, Stuttgart.

Welsch, W., 1979. A geodetic micro-network for monitoring tectonic movements in a Peruvian earthquake fault zone. In: A. Vogel (Editor), Terrestrial and Space Techniques in Earthquake Prediction Research, 1978. Vieweg, Braunschweig.